BEAM HALO FORMATION FROM SPACE-CHARGE DOMINATED BEAMS IN UNIFORM FOCUSING CHANNELS

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Abstract

In space-charge dominated beams the nonlinear space-charge forces produce a filamentation pattern, which results in a 2-component beam consisting of an inner core and an outer halo. The halo is very prominent in mismatched beams[1-3] and is a concern because of the potential for accelerator activation. We present new results about beam halo and the evolution of space-charge dominated beams from multiparticle simulation of initial laminar beams in a uniform linear focusing channel, and from a model consisting of single particle interactions with a uniform-density beam core. We study the energy gain from particle interactions with the space-charge field of the core[4], and we identify the resonant characteristic of this interaction as the basic cause of the separation of the beam into the two components[5]. We identify three different particle-trajectory types, and we suggest that one of these types may lead to continuous halo growth, even after the halo is removed by collimators.

I. PHASE-SPACE DYNAMICS FROM MULTIPARTICLE SIMULATION

We use multiparticle simulation to study a round continuous beam in a uniform linear focusing channel with purely radial focusing. For the studies described in this paper, the computer code has been run with 3000 simulation particles through 56 steps per plasma period. We have chosen to study the dynamics of initially Gaussian, laminar (zero-emittance) beams, where the initial density distributions are truncated at three standard deviations. In Fig. 1a we show the radial or r - r' phase space at 20 plasma periods for an initial mismatch parameter M = 1.5, where the mismatch parameter gives the ratio of the initial beam radius to the matched radius. We assign an initial positive radius to all particles, but if during the simulation a particle crosses the axis, we change the sign of the radius before plotting a point in r-r' space. Fig. 1a shows the inner core and the outer halo filament as distinct structures. At present there is no established criterion for defining the halo. We define the core and the halo by choosing a core ellipse in r-r' space with the same Courant-Snyder ellipse parameters as the rms ellipse with an emittance that we think encloses most of the core and excludes most of the halo.

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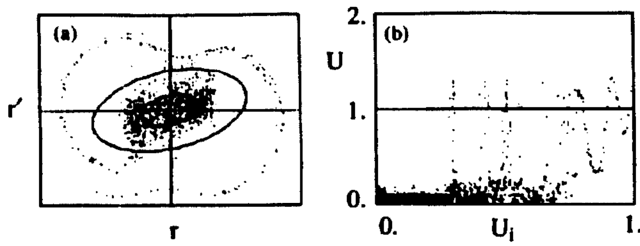

Figure 1. Results at 20 plasma periods from multiparticle simulation of an initial Gaussian-density laminar beam in a uniform linear focusing channel. The initial rms size in x is larger than the matched value by a factor M = 1.5. We show a) r - r' phase space, b) particle energy U versus the initial particle energy U_i.

Fig. 1b shows the distribution at 20 plasma periods of zero-current particle energy U versus initial U_i. The core appears as the concentration of particles at small U. All particle energies are normalized so that the particle with the maximum initial energy (largest initial radius) has $U_i = 1$. Fig. 1b shows that after 20 plasma periods, the beam does contain particles with U > 1, which have more energy than the most energetic initial particle. The halo is mostly populated by the particles that have large initial energy, i.e., have large initial radius. However, a small fraction of particles with small initial radius (only one such particle is present in the 3000 particle run and it is not visible in Fig. 1) also populate the halo, and the fraction of these halo particles with small initial energy increases rapidly with increasing mismatch parameter.

The characteristics of the beams after 20 plasma periods are shown in Table 1. All particle energies in Table 1 are normalized in the same way, to unity for the initial matched beam case. Column 1 shows the initial mismatch parameter M. Column 2 shows the values of the core emittance to rms emittance ratio. Column 5 shows that the maximum particle energy increases from the initial value by 10% for the matched case, and increases by 50% for a mismatch of M = 2.5. The last two columns show the percentage of final particles in the halo and the percentage of final particles with energy that exceeded the maximum initial particle energy. We see again that the percent of particles in the halo increases rapidly with increasing mismatch, and reaches a value of over 30% for an initial mismatch parameter of 2.5. We are unable to define the halo unambiguously for M=1.

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Table 1
Beam Halo Characteristics at 20 Plasma Periods for Beams with Different Initial Mismatch

M	e_{core}/e_{rms}	$U_{i,max}$	U_{max}	$U_{max}/U_{i,max}$	% in halo	% U > 1
1.0	—	1.00	1.10	1.10	—	0.1
1.5	7	2.25	2.97	1.32	5.4	2.2
2.0	3	4.00	5.90	1.48	16.	1.7
2.5	2	6.25	9.38	1.50	31.	2.0

II. PHASE-SPACE DYNAMICS FROM UNIFORM-DENSITY CORE MODEL

A. Energy Transfer Between a Single Particle and an Oscillating Core.

To gain some physical insight into what we observe in the simulation, we consider the model of a zero-emittance, uniform-density beam core of radius R, propagating in a uniform linear focusing channel. The transverse equation of motion of the beam radius is given by the envelope equation

$$\frac{d^2R}{dt^2} + \omega_0^2 R - \frac{K}{R} = 0,$$

where ω_0 is the natural frequency at zero current of a single particle, and nonrelativistically $K = qI/2\pi\varepsilon_0 mv$, where q, m, and v are the charge, mass, and axial velocity of the particles, I is the beam current, and ε_0 is the permeability of free space. For the matched beam $d^2R/dt^2 = 0$, and the matched beam radius is $R = R_0 = \sqrt{K} / \omega_0$. The transverse equation of motion of a single test particle is

$$\frac{d^2x}{dt^2} + \omega_0^2 x - F_{SC} = 0,$$

where F_{SC} is the space charge force, given for a uniform density by

$$F_{SC} = \begin{cases} Kx / R^2, & x < R. \\ K / x, & x > R. \end{cases}$$

The I=0 particle energy is not constant because of the space-charge repulsion of the core. For a matched core radius, there is no net change in energy averaged over a complete period of particle motion. For an oscillating, mismatched core there can be a net change in particle energy. For a small, radially symmetric core mismatch dR about the equilibrium radius R_0, we can write for x < R,

$$\frac{d^2x}{dt^2} + A\Omega^2 x \cos\Omega t \cong 0,$$

where $\delta R/R_0 = A \cos \Omega t$ and $\Omega = \sqrt{2}\,\omega_0$. This approximate result is a special case of the Mathieu equation, which suggests periodic solutions in x for particle frequencies below half the core frequency Ω. When the particle frequency is half the core oscillation frequency, we expect a resonant growth of the x amplitude. The resonant condition requires a constant phase relationship between the particle and the core. We expect that the effect of the nonlinearity outside the core is to create a self limit to the resonance because of the change of frequency with amplitude, which causes a loss of phase coherence. The core oscillation causes a rate of energy gain for a particle with velocity $\dot{x}$ within the beam core, given by

$$\frac{dU}{dt} = \vec{F} \bullet \dot{\vec{x}} = -A\Omega^2 x \dot{x} \cos\Omega t.$$

A particle that passes through the beam core can either gain energy, loose energy, or have no net energy change, depending on the relative phases of the particle displacement, the transverse velocity, and the core radius oscillation.

B. Trajectory Classification

A study of the trajectories of individual particles has been helpful for understanding the dynamics of the halo formation. The uniform-core model described above provides an important reference point for such a study. We have numerically integrated the trajectories of particles through the field of the uniform core for the M = 1.5 case, launching particles with different initial x values, and with $R = 1.5$, $\dot{R} = 0$, and $\dot{x} = 0$. We find three distinct classes of trajectories, which for the M = 1.5 case, we describe in terms of the initial x displacement as follows: a) For x < 1.5 the particles oscillate in phase with the core radius about their own equilibrium radius. These are stable radial plasma oscillations within the core, and we refer to these orbits as plasma trajectories. b) For x > 2.0 the particles oscillate about the origin with an orbit in phase space that looks like an ellipse that is pinched inward along the velocity axis (the pinching is caused by the space-charge force of the core). The amplitudes are variable and each orbit is confined to a narrow band in phase space. These particles occupy the halo and we call these orbits betatron trajectories. c) Finally, for 1.5 < x < 2.0 the particles execute a more complex motion. They may initially spend part of the time executing plasma-like oscillations within the core, after which they move into the proper phase relationship with respect to the core oscillations such that they can gain energy and move into an outer betatron-like orbit in the halo. These particles can also reverse the pattern and return from the outer betatron-like orbit in the halo to a plasma-like orbit within the core. We refer to these as hybrid trajectories, and we find that these particles are strongly affected by resonant energy transfer with the oscillating core, which can cause either energy loss into the core, or energy gain into the halo. Figure 2 shows examples of these trajectories in phase space traced for 20 core oscillation periods using the uniform core model, with $\omega_0 = 1$, and K = 1, launched with the following initial conditions: $R = 1.5$, $\dot{R} = 0$, and $\dot{x} = 0$, and a) x = 1.0, b) x = 1.55, c) x = 1.65, and d) x = 2.2. The outlines of the core-radius oscillation are shown on each of the plots, extending from $x = \pm 0.5743$ to $x = \pm 1.50$. These examples show a pure plasma trajectory in Fig.2a, a hybrid trajectory that has not yet left the core in Fig. 2b, a hybrid trajectory in the halo in Fig. 2c, and a periodic betatron trajectory in the halo in Fig. 2d.

The discovery of the hybrid trajectories is important because of the implications for the effectiveness of collimation. Suppose at a given time the beam is collimated to remove the existing halo. Any such collimation procedure, even if carried out under the most ideal of circumstances, would be able to remove only the particles with betatron trajectories and the particles with hybrid trajectories, which at that time populate the halo. Any hybrid orbits that are within the core at the time of collimation may be expected to gain energy at some later time and repopulate the halo. Therefore, while collimation does some good, it's effectiveness would be limited by the percentage of hybrid particles that would repopulate the core within the time scale of interest. Although this may appear to be a serious limitation of the collimation, it appears in the uniform beam model that the hybrid amplitudes in the halo are bounded. For this example collimation at a radius of 1.5 units could still be effective in limiting all hybrid amplitudes to less than $x = \pm 2.7$.

III. ACKNOWLEDGMENTS

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IV. REFERENCES

[1] A. Cucchetti, M. Reiser, and T. P. Wangler, "Simulation Studies of Emittance Growth in RMS Mismatched Beams," Proceedings of 1991 Particle Accelerator Conference," San Francisco, California, May 6-9, 1991, IEEE Conf. Record 91CH3038-7, p 251.

[2] T. P. Wangler, "Emittance Growth from Space-Charge Forces," High-Brightness Beams for Advanced Accelerator Applications, AIP Conf. Proc. **253**,21(1991).

[3] T. P. Wangler, "Beam Halo in High-Intensity Beams", presented at the Computational Accelerator Physics Conference (CAP93), Pleasanton, CA, February 22-26,1993.

[4] R. A. Jameson has developed this idea independently. See R. A Jameson, "Beam-Halo from Collective Core/Single-Particle Interactions", Los Alamos Report LA-UR-93-1209, March, 1993.

[5] R. L. Gluckstern suggested in an informal note "Possible Model for Halo Formation", March, 1993, the idea that a resonant nonlinear interaction between individual particles and an oscillating core could lead to a depopulation of a region of phase space and that would explain the absence of particles between the core and the halo.

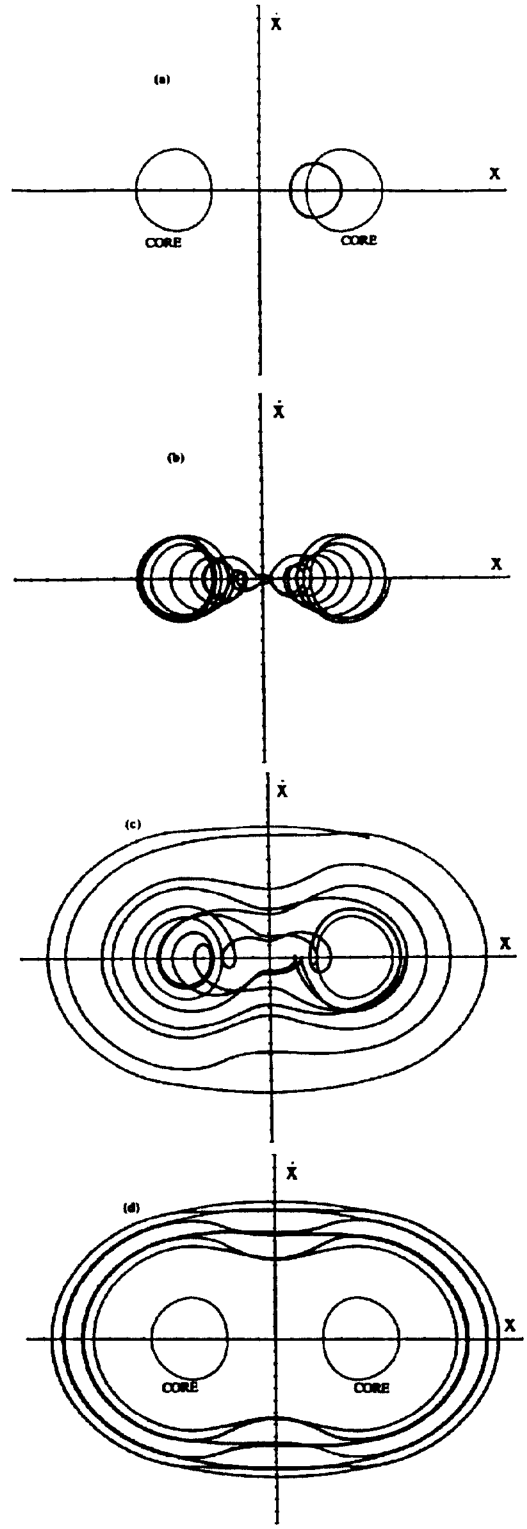

Fig. 2. Trajectories in phase space traced for 20 core oscillation periods using the uniform core model, with $\omega_0 = 1$, and K = 1, launched with the following initial conditions: R=1.5, $\dot{R} = 0$, and $\dot{x} = 0$ and a) x = 1.0, b) x = 1.55, c) x = 1.65, and d) x = 2.2. The outlines of the core-radius oscillation are shown on each of the plots, extending from x = - 1.50 to x = - 0.5743 and from x = 0.5743 to x = 1.50.